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A Fertility Distribution Function for Rural India (States)-1971*

NOT much attention has been paid by demographers in India either to determine suitable fertility distribution functions or, at least, to recognise their practical usefulness in situations where the basic data are defective and age-specific fertility rates computed therefrom are subject to errors resulting from age mis-statements, and omission of births. The civil registration system which has been in vogue for a very long time provides data required for estimating age specific fertility rates (ASFR's), but because of serious deficiencies in the data, these ASFR's cannot be considered very reliable. In the post-Independence period, a few demographic sample surveys have been conducted but they are more in the nature of local and exploratory studies meant for getting an insight into ascertainable socio-economic factors affecting fertility. There have also been quite a few surveys with countrywide coverage, for example, National Sample Surveys (NSS) from the 14th (1958-59) to the 28th Round (1973-74). The demographic information furnished by the NSS, though of better quality than the civil registration data, is still found to underestimate fertility. Being essentially retrospective in character, these data suffer from errors of imprecision in the reference period and recall lapse, in addition to age mis-statements.

The office of the Registrar General has been operating the Sample Registration Scheme (SRS) for several years now. The estimates of vital rates thrown

*The views expressed in this paper are entirely Authors' own.

up by this scheme are adjudged to be more reliable than any of those provided by other sources. In view of this, this paper attempts to obtain a fertility distribution function for Rural India (States) in 1971 on the basis of census (1971) and SRS data. Since the rural population of States account for 99.44 percent of the total rural population, the fertility distribution derived in this paper may be regarded as valid for the entire rural population of the country for the year 1971.

Methodology

The methodology adopted is to first graduate the cumulative fertility distribution by the Method of Least Squares so that the age-specific fertility distribution function may then be obtained by differentiating the cumulative function. The general equation of the cumulative fertility distribution function may be taken in the following polynomial form of n-th degree

$$Y=(x-a)^2 \{A + Bx + Cx^2 + \dots + Kx^{n-2}\}$$

where 'y' indicates the total number of live births to a woman aged 'x' years, V the lower limit of the reproductive span and A, B, C, . . . are the parameters. As may be seen, the only condition placed on the cumulative fertility distribution function is that its value is equal to zero at $x = a$, the beginning of the reproductive span. For smoothening the cumulative fertility distribution, the observed cumulative values of ASFR's for rural India (States) for 1971 at the seven exact values of $x = 20, 25, 30, 35, 40, 45,$ and 50 years have been taken from the empirical data described later in the paper. The reason for taking the cumulative totals of ASFR's to these exact seven ages for smoothening cumulative fertility distribution is that errors due to mis-statements of age are likely to get reduced due to cyclic components which might not be the case when age specific fertility rates are grouped by age intervals. The parameters A, B, C, . . . etc. of the polynomial are then calculated by fitting the curve through observed values at these points in such a way that sum of squares of the difference between the observed and the theoretical measures becomes minimum.

Different polynomials, with degrees—five, four and three have been tried to get the best fit. These functions are different not only in respect of degree but also in respect of the value of 'a', the lower limit of the reproductive span.

the equations of these models are described briefly here.

$$Y = (x - a)^2 \{A + Bx + Cx^2 + Dx^3\}$$

is a polynomial of 5th degree in V and in this paper, $a = 15.0$ years has been tried. In the 4th degree polynomial represented by

$$Y = (x - a)^2 \{A + Bx + Cx^2\}$$

six different values of ' a ' viz., 15.0, 15.3, 15.4, 15.5, 16.0 and 17.0 years were employed. In one model represented by a 3rd degree polynomial of the form

$$Y = (x - a)^2 \{A + Bx\}$$

we have studied the case when $a = 15.0$ years.

Source of Data

For estimation of ASFR's for rural India (States) 1971 data from various sources for seven age-groups 15-19, 20-24, . . . , 45-49 have been utilised. The age-wise and sex-wise distribution of the population of India for 1971 is provided by the Registrar General's Office. SRS has also furnished the rural crude birth rates for individual States for three consecutive years 1970, 1971 and 1972. The average crude birth rates for the three-year period 1970-72 have been taken as more reliable estimate of the CBR of the individual States for 1971 as in this process the annual fluctuations are smoothed out to some extent. The number of total births for each of the 12 States in respect of which the SRS has furnished the ASFRs in addition, is then worked out on the basis of the CBR thus derived and the total population provided by the census 1971 for these States. These births are then distributed in the seven age-groups viz., 15-19, 20-24, 25-29, 30-34, 35-39, 40-44 and 45-49 as per the rural pattern of ASFR's provided by the SRS for these States for an earlier year viz., 1969. Thus it is assumed that even in 1971, the inter-relationship between ASFR's for different age-groups has remained the same as provided by the SRS for 1969 for rural areas for these individual States. For the remaining States the total births worked out on the basis of CBR and census population 1971 (rural) have been distributed among the seven age groups as per the pattern of ASFR's of the States which could be presumed to have similar fertility patterns. Thus, for the States for which the SRS has not provided

the ASFR, the 'indirect' method has been used so that ASFR's for Rural India (States) 1971 could be worked out. The only exception to this procedure is in respect of the States of West Bengal and Bihar in which cases the CBR's provided by the SRS are considered to be gross under-estimates. In order to arrive at more reliable estimates for these two States the linear relationship between the CBR (SRS) and child-woman ratio (census) was examined. In working out the linear regression equation the data for West Bengal and Bihar were not included. The relationship was found to be of the form

$$U = 17.8989 + 0.0232 v$$

where

$$U = \text{CBR and } v = \text{child-woman ratio.}$$

The linear regression has provided a reasonably good fit between U and v and the coefficient of regression was found to be highly significant at 1% level. On the basis of this regression analysis, the CBR's for West Bengal and Bihar were estimated and the estimates were considered to be more reliable. These have been preferred to the CBR's provided by the SRS for these two States. The data are shown in Annexure II.

Findings

The empirical data indicate a total fertility rate of 5.60 for rural India (States) in 1971. Further, 'observed' ASFR data for rural India, 1971 show a flat topped curve in the age group 20-29 with very insignificant difference between the ASFR's for 20-24 years (0.2540) and 25-29 years (0.2570). Even for the next age group 30-34 the fall in the ASFR from its high values in the previous age interval is only 12%. These rather unusual features of the observed fertility distribution function were considered to be mainly due to the inherent defects in the data. Out of the models fitted to the observed data, it is found that the 4th degree model with the following equation gives the best fit as in this case the sum of the squares of differences between the observed and the computed values is the minimum.

$$Y = (x - 15.40)^2 (0.04718482 - 0.00145819x + 0.00001214x^2).$$

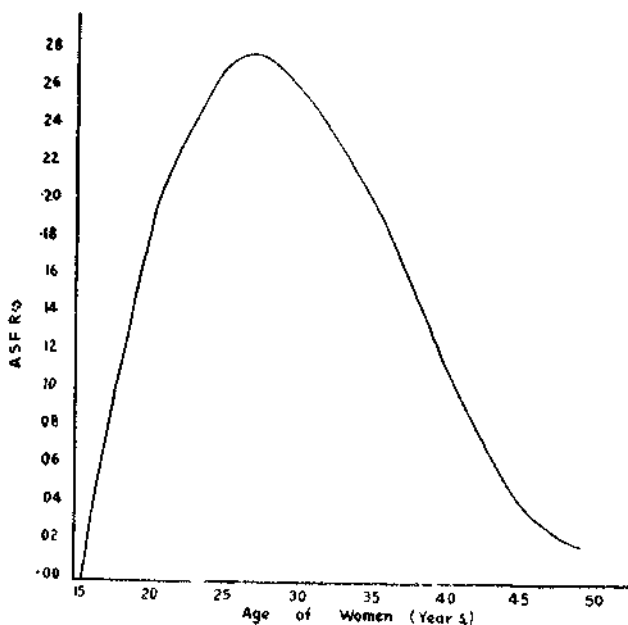
By differentiating the equation of this model, we find the equation of the fertility distribution function as

$$f(x) = 0.000\ 048\ 56 \{x - 15.40\} (48.89 - x)^2 + 15.5679\}$$

where $f(x)$ is the ASFR at age x and $15.40 \leq x \leq 48.89$. Thus the fertility distribution function is valid only for the age interval from 15.40 to 48.89 years and its value may be taken as equal to zero outside the range of 15.40 to 48.89. The mode of the function is 26.80 years and the mean of the distribution 29.48. The values of the function at a few specific ages are as under:

x	15.40	20.0	25.0	30.0	35.0	40.0	45.0	48.89
$f(x)$	0	.1899	.2730	.2639	.1983	.1127	.0438	.0248

Age Specific Fertility Distribution Curve For Rural India-1ST)



The ASFR's for the seven age groups as obtained from this model are indicated in Annexure I where similar data derived from other models as well as the observed ASFR's are also indicated.

It may be pointed out in this connection that W. Brass has suggested a model fertility distribution function as:

$$f(x) = C(x - a) (a + 33 - x)^2; a \leq x \leq a + 33$$

where $f(x)$ is the age-specific fertility rate of a women aged x years, 'a' is the start of reproductive period and 'C' is a constant which varies with the level of fertility. $f(x)$ is taken as zero when 'x' is outside the range of 'a' to $a + 33$ years. The function is therefore equal to zero at $x = 'a'$ and $x = 'a + 33'$. Between these values the curve has a sharp rise in the beginning with a peak at 'a + 11' (mode) years. After attaining the peak the curve has a more gradual fall to the end of reproduction at 'a + 33' years. The mean of the distribution is $a + 13.2$ years. The mean and the mode of the curve are nearer to the lower limit of reproductive period than to the upper limit.

The fertility distribution function

$$f(x) = .00004856 (x - 15.40) \{(48.89 - x)^2 + 15.5679\}$$

obtained for Rural India (States)— 1971 could be written in the general form

$$f(x) = C(x - a) \{(b - x)^2 + \lambda\}$$

which is distinct from the Brass's function stated above but the similarity in form is easily noticed.

The Brass function may be regarded as a particular case of the general function given by

$$\Phi_{m,n}(x) = C(x - a)^m (b - x)^n,$$

where $a \leq x \leq b$; and m and n are positive integers. For this general function we have

$$C = K \tau(m+n+2) / (b-a)^{m+n+1} \cdot \tau(m+1) \cdot \tau(n+1)$$

where K = total fertility rate

$$\text{Mean} = a + \frac{m+1}{m+n+2} (b-a)$$

and
$$\text{Mode} = \frac{mb+na}{m+n}.$$

Hence for Brass's function which is $\phi_{1,2}(x)$,

$$C = \frac{12K}{(b-a)^4}$$

$$\text{Mean} = a + \frac{2}{5} (b-a), \text{ and}$$

$$\text{Mode} = \frac{b+2a}{3}.$$

Writing the new fertility function in the form

$$f(x) = C(x-a) \{(b-x^2) + \lambda\},$$

C may be shown to be equal to

$$\frac{12K}{(b-a)^4 + 6\lambda(b-a)^2}$$

which is less than the corresponding value of C for Brass's curve, where K is total fertility rate.

$$\text{Mean} = \frac{(3a+2b) + 10\lambda \left(\frac{2b+a}{(b-a)^2} \right)}{5 \left\{ 1 + \frac{b\lambda}{(b-a)^2} \right\}}.$$

It may be noticed that the mean will be equal to $\frac{3a+2b}{5}$ when $\lambda = 0$, in which case this equation gets reduced to that of Brass model. For the fertility function for rural India 1971 which has been derived in the paper, it will

be noticed that the values of a and b are 15.40 and 48.89 years respectively. Corresponding to these values, the mean of the age specific distribution works out to 29.48 years which is 0.68 year higher than the mean of the Brass model having the same values for the limits of the reproductive span. Mode in this case may be shown to be equal to

$$\frac{2b + a}{3} - \frac{\sqrt{4(b-a)^2 - 3\lambda}}{3} = \frac{b + 2}{3} + \frac{\lambda}{2(b-a)}$$

approximately i.e. $> b+2a / 3$ i.e. the mode of the new distributing function is also somewhat higher than the mode for the Brass function. It may be noticed that the function developed in this paper is actually a generalised form of the Brass function: when $\lambda = 0$, the two functions become identical.

From the angle of demographers, the general properties of the new fertility function which is represented by a single peaked curve in the reproductive range, are broadly similar to those of the Brass model. However, unlike the Brass model, the fertility function developed in this paper does not exactly vanish at the upper limit of the reproductive period i.e. 48.89 years but indicates a very small value of the order of 0.0248.

Acknowledgement

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ANNEXURE I

<i>Models</i>	<i>Cumulative Fertility Function</i>	<i>Age Specific Fertility Function</i>
I	$y = (x - 15)8 (.0296578 - .00033215 x - .0000119x^2 + .000000169 x^3)$	$f(x) = .000000845 (x - 15) (76092.13 + x (x - 32.665)^2 - 1823.75)$
II	$y = (x - 15)^2 (.0358854 - .0009247 x + .00000586 x^2)$	$f(x) = .00002344 (x - 15) ((62.925 - x)^2 - 305.92)$
III	$y = (x - 15.30)^2 (.04406574 - .00131007 x + .00001039 x^2)$	$f(x) = .00004156 (x - 15.30) ((51.1085 - x)^2 - 9.2028)$
IV	$y = (x - 15.40)^2 (.04718482 - .00145819 x + .00001214 x^2)$	$f(x) = .00004856 (x - 15.40) ((48.89 - x)^2 + 15.5679)$
V	$y = (x - 15.50)^2 (.05052818 - .00161768 x + .00001403 x^2)$	$f(x) = .00005612 (x - 15.50) ((47.113 - x)^2 + 27.8777)$
VI	$y = (x - 16)^2 (.071332361 - .002622313 x + .0000260312 x^2)$	$f(x) = .00010412 (x - 16) ((41.78 - x)^2 + 27.59)$
VII	$y = (x - 17)^2 (.15119975 - .00660695 x + .0000747 x^2)$	$f(x) = .0002988 (x - 17) ((37.42 - x)^2 - 12.3125)$
VIII	$y = (x - 15)^2 (0.0292929 - .0005145 x)$	$f(x) = .0015435 (x - 15) (42.956 - x)$

A.S.F.R. 's

Annexure I (contd.)

Age Group	Observed								
	1971 Rural India	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII	Model VIII.
15 - 19	.0954	.0980	.0987	.0973	.0968	.0963	.0938	.0881	.0950
20 - 24	.2540	.2331	.2299	.2378	.2408	.2440	.2634	.3306	.2336
25 - 29	.2571	.2784	.2752	.2749	.2748	.2746	.2736	.2647	.2950
30 - 34	.2264	.2465	.2521	.2392	.2342	.2288	.1952	.0594	.2792
35 - 39	.1630	.1624	.1782	.1619	.1556	.1488	.1060		.1860
40 - 44	.0832	.0640	.0711	.0742	.0753	.0765	.0842		.0350
45 - 49	.0423	.0017	.0023	.0073	.0240	.0009	

ANNEXURE II

STATE-WISE NUMBER OF BIRTHS IN 1971 AMONG WOMEN IN AGE GROUPS 15-49 YEARS AND AGE

- SPECIFIC FERTILITY RATES 1971 FOR RURAL INDIA (STATES)

	<i>States</i>	<i>Age Groups</i>							<i>Total</i>
		<i>15-19</i>	<i>20-24</i>	<i>25-29</i>	<i>30-34</i>	<i>35-39</i>	<i>40-44</i>	<i>45-49</i>	
1.	Andhra Pradesh	192444	325544	329349	209618	131226	50749	14118	1253048
2.	Assam (including Meghalaya)	82771	159667	155632	99777	45776	14790	2529	560942
3.	Bihar	152304	428512	398180	362033	237990	115748	54948	1749715
4.	Gujarat	61489	246744	216720	160352	85585	32743	6681	810314
5.	Haryana	29205	86461	84029	70644	41711	24070	6881	342966
6.	Himachal Pradesh	12422	27661	25685	22145	14553	7122	3378	112966
7.	Jammu & Kashmir	15389	31133	37407	23632	15685	5917	2388	131551
8.	Kerala	55528	184685	132697	99508	68551	21235	3116	565320
9.	Madhya Pradesh	122266	334561	334650	290932	191179	91722	43484	1408794
10.	Maharashtra	98275	297671	326154	233538	134208	46556	12236	1148638
11.	Manipur	2785	9571	8420	5545	3818	1086	159	31384
12.	Karnataka	113372	198303	199621	135992	76202	25909	6783	756182

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13	Orissa	67457	159513	173760	151857	99812	46953	22298	721650
14	Punjab	12541	98198	100870	76421	46716	24196	4829	363771
15	Rajasthan	85729	237989	229253	165899	111700	50116	21282	901968
16	Tamil Nadu	90947	270847	296543	181030	93564	26266	6431	965628
17	Tripura	3817	13992	14820	9486	6532	1828	267	50742
18	Uttar Pradesh	303256	829115	816105	718175	471974	235612	111855	3486092
19	West Bengal	134701	300611	295869	245109	161065	74420	35307	1247082
	Total births	1636698	4240778	4175764	3261693	2037847	897038	358907	16008725
	A. S. F. R.- 1971 (Rural)	0.0954	0.2540	0.2571	0.2264	0.1630	0.0832	0.0423	1.1214

Demography India